

Name: \_\_\_\_\_

Math 260

Start Time: \_\_\_\_\_

Quiz 5 (40 min)

End Time: \_\_\_\_\_

Date: \_\_\_\_\_

1. (1 point) Complete the following definition:

Definition: The square matrix  $A$  is invertible if...

2. (2 points) Prove the uniqueness of the inverse of a matrix. That is, prove the following statement:

If  $B$  and  $C$  are inverses of a square matrix  $A$ , then  $B = C$ .

3. (2 points) Use the matrix inversion algorithm to find  $A^{-1}$  if  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 5 & -1 \end{bmatrix}$  (calculator OK!)

4. (3 points) Write  $A = \begin{bmatrix} 2 & 5 \\ -3 & -2 \end{bmatrix}$  as a product of elementary matrices.

5. (2 points) Give an example of a square matrix that is not invertible and explain why it is not invertible

Extra Credit

1. (2 points) Prove: If  $A$  and  $B$  are  $n \times n$  invertible matrices, then  $AB$  is invertible and  $(AB)^{-1} = B^{-1}A^{-1}$

2. (2 points) Prove: If  $A$  is an  $n \times n$  invertible matrix, then  $A^T$  is invertible and  $(A^T)^{-1} = (A^{-1})^T$

3. (4 points) True or False? (For each of the following, assume  $A$  and  $B$  are square matrices of the same size)

a) If  $A$  is an invertible matrix, then when reducing it to reduced row-echelon form, you get the identity matrix.

b) If the matrix  $A$  has a right inverse, then it is invertible.

c) If the system of equations  $A\vec{x} = \vec{b}$  has a solution for any choice of  $\vec{b}$  then  $A$  has an inverse.

d) If  $A$  and  $B$  are invertible then so is  $A + B$